# Ramanujan School of Mathematics

Class Test on Complex Numbers

April, 2019

Total time is 2 hours. Total marks is 50. Attempt as many as you can.
Answers without proper explanations will fetch no mark.

1. (10 marks) Suppose that $z_1, \cdots, z_n$ and $w_1, \cdots, w_n$ are complex numbers with $|z_i| \leq 1$ and $|w_i| \leq 1$ for each $i = 1, 2, \cdots, n$. Show that,

$$|z_1 z_2 \cdots z_n - w_1 w_2 \cdots w_n| \leq |z_1 - w_1| + |z_2 - w_2| + \cdots + |z_n - w_n|.$$

2. (10 marks) On the sides $AB, BC, CD, DA$ of a quadrilateral $ABCD$, we construct squares (exterior to the quadrilateral) with centers $O_1, O_2, O_3, O_4$ respectively. Prove that $O_1O_3 \perp O_2O_4$ and $O_1O_3 = O_2O_4$.

3. (10 marks) Find all positive real numbers $x, y$ satisfying the system of equations:

$$\begin{aligned} \sqrt{x}\Big(1 + \frac{1}{x+y}\Big) &= \frac{3}{2}, \\ \sqrt{y}\Big(1 - \frac{1}{x+y}\Big) &= \frac{1}{2}. \end{aligned}$$

4. (10 marks) Suppose that $A_1A_2 \cdots A_n$ is a regular polygon with $n$ sides ($n > 2$), inscribed in a circle of radius $r$. Find the product of the distances of $A_1$ from all the other vertices. That is, find $A_1A_2 \times A_1A_3 \times \ldots \times A_1A_n$.

5. (10 marks) A sequence $(a_1, b_1)$, $(a_2, b_2)$, $(a_3, b_3)$, $\ldots$ of points in the coordinate plane satisfies

$$(a_{n+1}, b_{n+1}) = (\sqrt{3}a_n - b_n, \sqrt{3}b_n + a_n) \text{ for each } n = 1, 2, 3, \cdots.$$

Suppose that $(a_{100}, b_{100}) = (18, 20)$. What is $a_1 + b_1$?

*Do not cheat to yourself. All the best!*

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