Ramanujan School of Mathematics

Class Test on Complex Numbers

March 2020

Total marks: $10 \times 5 = 50$ Time: 2 hours.

Attempt all the questions. Answers without proper explanations will fetch zero. Show all your rough work – partial solutions may be rewarded. You can use any theorem/result without proving it again; but you have to state it properly.

1. Suppose that the roots of the equation $x^4 + ax^3 + bx^2 + ax + c = 0$ are $\alpha_1, \alpha_2, \alpha_3,$ and α_4. Show that, $(\alpha_1^2 + 1)(\alpha_2^2 + 1)(\alpha_3^2 + 1)(\alpha_4^2 + 1) = (1 - b + c)^2$.
2. Let $\triangle ABC$ be an equilateral triangle with the circumradius equal to 1. Prove that for any point P on the circumcircle, we have $PA^2 + PB^2 + PC^2 = 6$.
3. Determine the value of $\cos \frac{\pi}{11} + \cos \frac{3\pi}{11} + \cos \frac{5\pi}{11} + \cos \frac{7\pi}{11} + \cos \frac{9\pi}{11}$.
4. Show that for any positive integer n, the following identity holds:
$$\left(\binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \cdots \right)^2 + \left(\binom{n}{1} - \binom{n}{3} + \binom{n}{5} - \cdots \right)^2 = 2^n.$$
5. An ant is moving on the coordinate plane. Initially it was at $(6, 0)$. Each move of the ant consists of a counter-clockwise rotation of 60° about the origin followed by a translation of 7 units in the positive x-direction. If the position of the ant after 2020 moves is (p, q), find the value of $p^2 + q^2$.

Do not cheat to yourself. All the best!

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