Ramanujan School of Mathematics

Class Test on Complex Numbers

Time allotted: 2 hours Total points: 40

Attempt all the questions. You can use any result discussed in the class, but you have to state it properly. Since it is a 'take-home' exam, I can only request you to take the test honestly and abide by the time limit. Do not cheat to yourself. All the best!

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1. Suppose that A, B, C are any three angles satisfying

$$\cos 2A + \cos 2B + \cos 2C = \cos(A+B) + \cos(B+C) + \cos(A+C),$$
$$\sin 2A + \sin 2B + \sin 2C = \sin(A+B) + \sin(B+C) + \sin(A+C).$$

Show that $\cos A + \cos B + \cos C = \sin A + \sin B + \sin C$.

2. Let $ABCD$ be a square with centre O and let M, N be the midpoints of BO and CD respectively. Prove that $\triangle AMN$ is an isosceles right triangle.

3. Let n be any positive integer. Define

$$A = \binom{n}{0} - \binom{n}{3} + \binom{n}{6} - \cdots,$$
$$B = -\binom{n}{1} + \binom{n}{4} - \binom{n}{7} + \cdots,$$
$$C = \binom{n}{2} - \binom{n}{5} + \binom{n}{8} - \cdots.$$

Show that

(i) $A^2 + B^2 + C^2 - AB - BC - CA = 3^n$, (ii) $A^2 + AB + B^2 = 3^{n-1}$.

4. Let a, n be integers and let p be a prime such that $p > |a| + 1$. Prove that the polynomial $f(x) = x^n + ax + p$ cannot be factorized as the product of two non-constant polynomials with integer coefficients.

(Hint: First show that each root of $x^n + ax + p = 0$ must have modulus greater than 1.)