

Ramanujan School of Mathematics

Class Test on Complex Numbers

July 3, 2021

Time allotted: 1 hour

Total marks: $10 \times 2 = 20$

Attempt any TWO questions.

Show all your rough work – partial solutions may be rewarded. You may use any theorem/result without proving it again; but you have to state it properly.

1. Given a triangle ABC , construct two squares $ABMN$ and $ACPQ$ outwardly (such that the squares do not have any overlap with $\triangle ABC$). Let $AD \perp BC$ with D on BC , and E be the midpoint of NQ . Show that the points D, A, E are collinear.
2. Determine, with proof, the value of

$$\cos \frac{\pi}{7} - \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7}.$$

3. Suppose that $P(x), Q(x), R(x), S(x)$ are polynomials such that

$$P(x^5) + xQ(x^5) + x^2R(x^5) = (x^4 + x^3 + x^2 + x + 1)S(x)$$

holds for every $x \in \mathbb{C}$. Prove that $x - 1$ is a factor of $S(x)$.

4. Let n be any positive integer. Define

$$A = \binom{n}{0} - \binom{n}{3} + \binom{n}{6} - \dots, \quad B = -\binom{n}{1} + \binom{n}{4} - \binom{n}{7} + \dots,$$
$$C = \binom{n}{2} - \binom{n}{5} + \binom{n}{8} - \dots.$$

Show that, (i) $A^2 + B^2 + C^2 - AB - BC - CA = 3^n$, (ii) $A^2 + AB + B^2 = 3^{n-1}$.

Take the test honestly, do not cheat to yourself. All the best!